

Unit 5: Circular Functions ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL work as it is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

Radian Measure and Angles in Standard Position:

1. Define the following terms.

a) Standard Position Angle: _____

b) Initial Arm: _____

c) Terminal Arm: _____

d) Co-Terminal Angle _____

2. Convert each of the following into radians exactly (in terms of π).

a) 30° b) 240° c) 110° d) -10°

3. Convert the following to degrees exactly.

a) $\frac{\pi}{3}$

b) $\frac{5\pi}{6}$

c) $\frac{\pi}{10}$

d) $-\frac{5\pi}{9}$

4. Determine one positive and one negative co-terminal angle for the following angles.

a) 140°

b) $\frac{9\pi}{7}$

c) -78°

d) $-\frac{6\pi}{5}$

5. Determine the length of an arc in a circle of radius 4cm if it subtends an angle of 2rads .

6. Determine the radius length in a circle if an arc of length 5cm subtends an angle of 20° .

Lesson TwoSine, Cosine and Tangent:

1. Define Reference Angle:

2. Draw each angle in standard position and determine its reference angle.

a) $x = 70^\circ$

b) $x = 220^\circ$

c) $x = -100^\circ$

d) $x = 500^\circ$

e) $\theta = \frac{\pi}{7}$

f) $\theta = \frac{6\pi}{5}$

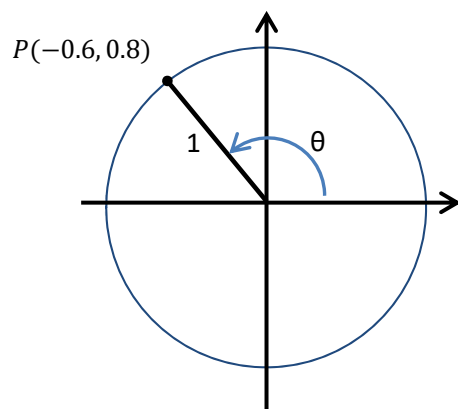
g) $\theta = -\frac{\pi}{4}$

h) $\theta = \frac{18\pi}{5}$

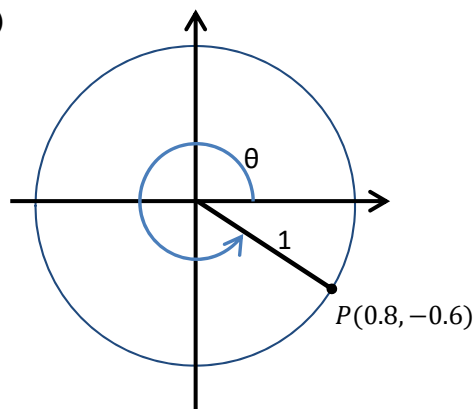
i) $\theta = -\frac{5\pi}{8}$

3. For each diagram determine $\sin\theta$, $\cos\theta$ and $\tan\theta$.

a)



b)



4. Point $A(-12, -9)$ is in Quadrant III and lies on the terminal arm of the standard position angle θ . Point P is the point of intersection of the terminal arm of θ and the unit circle. Determine the coordinates of P .

5. Given that the angle θ is in standard position with its terminal arm in quadrant two, find the exact value of $\cos\theta$ and $\tan\theta$ if $\sin\theta = \frac{1}{6}$

6. Find the exact value of $\sin \theta$, $\cos \theta$ and $\tan \theta$ if the terminal arm of the standard position angle θ contains the point

a) $(-2, -5)$

b) $(-3, 7)$

7. Given that $\cos \theta = \frac{2}{3}$ and that $\sin \theta < 0$, determine the exact value of $\sin \theta$.

8. Given that $\sin \theta = -\frac{1}{4}$ and that $\cos \theta < 0$, determine the exact value of $\tan \theta$

9. Given that $\sin \theta = a$, with $0 < a$ and that $\tan \theta < 0$, determine $\cos \theta$.

10. Given that $\tan \theta = -\frac{3}{2}$ with $\frac{\pi}{2} < \theta < \pi$, determine the exact value of $\cos \theta$.

11. If $\cos \theta = \frac{3}{4}$ with $0 < \theta < \frac{\pi}{2}$, then determine the exact value of $\sin(\pi + \theta)$

Lesson Three

Sine, Cosine and Tangent of Angles:

1. What is the purpose of the phrase “All students take calculus”? _____

2. Using your calculator, evaluate each of the following trigonometric ratios to the nearest thousandth.

a) $\sin 3.45$

b) $3 \tan 2.40$

c) $\frac{2}{\cos 4.67}$

d) $\csc 2.65$

3. Determine the exact value for the following questions (do not use a calculator).

a) $\sin \frac{\pi}{3}$

b) $\cos \frac{3\pi}{4}$

c) $-\tan \frac{5\pi}{6}$

d) $\sin \frac{7\pi}{4}$

e) $\cos \frac{3\pi}{2}$

f) $\csc \frac{\pi}{4}$

g) $3 \tan \frac{5\pi}{3}$

h) $\sin \left(-\frac{3\pi}{4} \right)$

i) $\cot \frac{5\pi}{6}$

j) $4 \sec \frac{11\pi}{6}$

k) $8\sec\left(-\frac{7\pi}{6}\right)$

l) $-2\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4}$

m) $2\csc\frac{5\pi}{2}$

n) $2\tan\frac{5\pi}{6} - \csc\frac{10\pi}{3}$

o) $\log_3\left(\tan\frac{\pi}{3}\right)$

p) $\log_3\left(\tan\frac{7\pi}{6}\right)$

q) $\log_2 \left(\sin \frac{3\pi}{4} \right)$

r) $\log_2 \left(\sec \left(-\frac{\pi}{3} \right) \right)$

4. Determine in which quadrant the standard position angle θ is located when:

a) $0 < \sin \theta$ and $\sec \theta < 0$

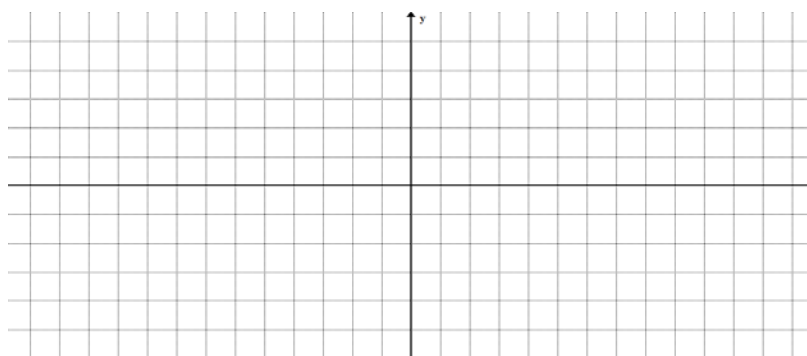
b) $\cot \theta < 0$ and $\csc \theta < 0$

Lesson Four

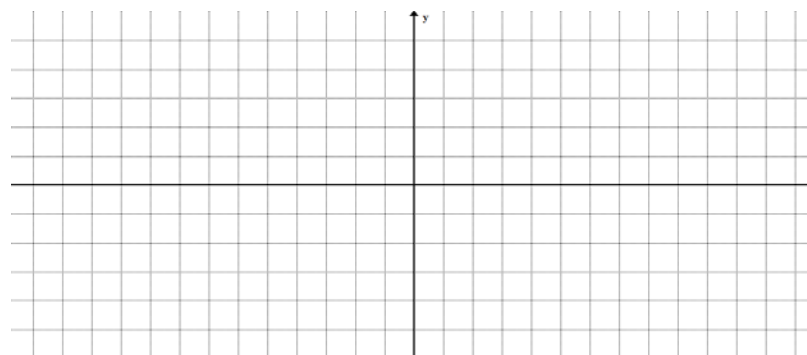
Graphing Sine and Cosine Functions:

1. Without using a calculator or making a table of values, graph the desired function over the interval $-2\pi \leq \theta \leq 2\pi$.

a) $y = \sin \theta$



b) $y = \cos \theta$



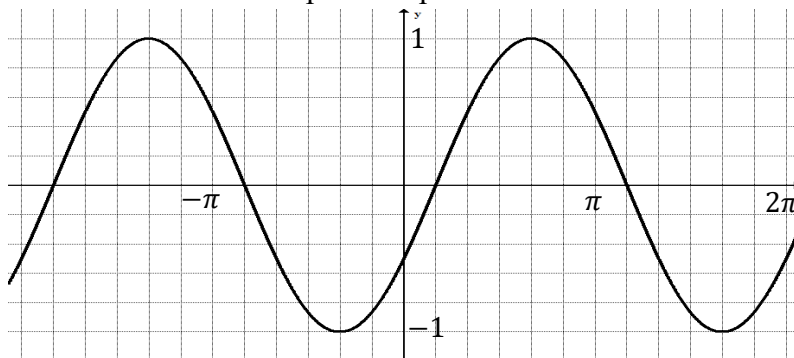
2. For the graph of $y = \sin \theta$ graphed on the interval $0 \leq \theta \leq 2\pi$
 - a) What is the maximum value of y ? For what values of θ does this occur?
 - b) What is the minimum value of y ? For what values of θ does this occur?
 - c) Determine the θ -intercepts.
3. For the graph of $y = \cos \theta$ graphed on the interval $0 \leq \theta \leq 2\pi$
 - a) What is the maximum value of y ? For what values of θ does this occur?
 - b) What is the minimum value of y ? For what values of θ does this occur?
 - c) Determine the θ -intercepts.

Lesson Five

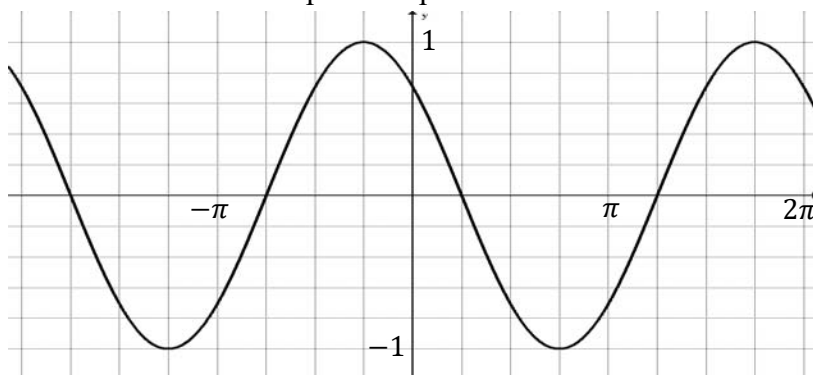
Transformations of Trigonometric Functions I:

1. Given the sinusoidal function $y = a \cos(x - c) + d$, discuss what the effect of each of the variables a , c and d have on the graph (Think of “trigonometric graphing” terms rather than “transformations” terms).
 - a) a : _____
 - b) c : _____
 - c) d : _____
2. The graph of y is transformed as described below. The equation of its image has the form $y = a \sin(x - c) + d$. Write the new equation for y by determining a , c and d .
 - a) Translate the graph 4 units right _____
 - b) Expand vertically by 3, then translate down 1 unit _____
 - c) Compress vertically by $\frac{1}{2}$, then translate $\frac{\pi}{7}$ units left _____

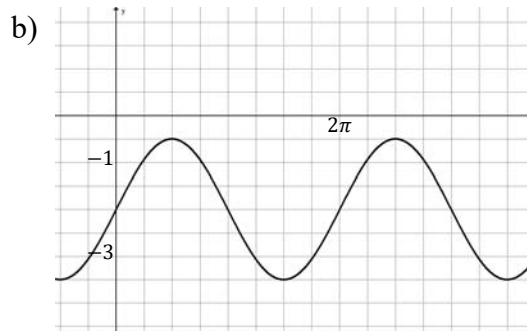
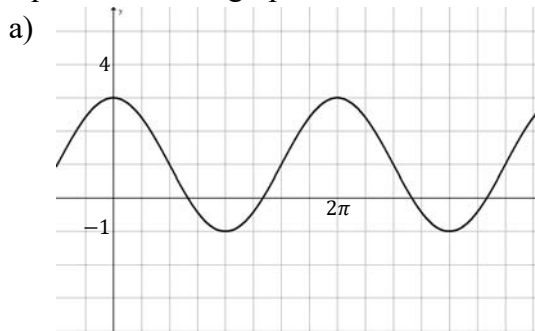
3. The graph below can be considered as a sine function. Determine the equation of this function with **smallest** possible phase shift.

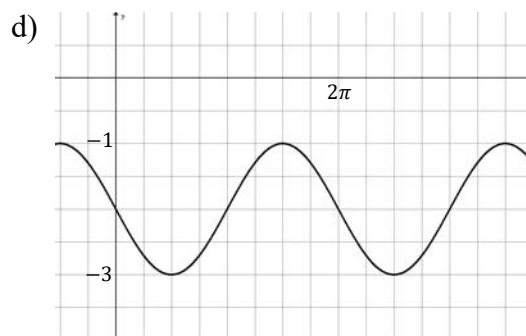
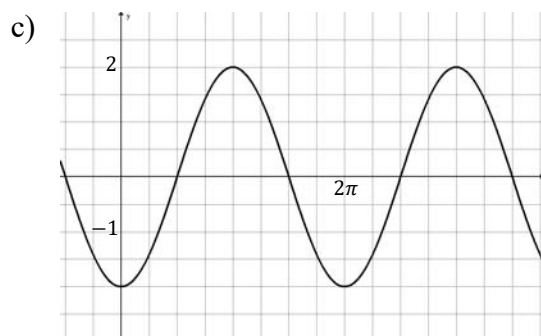


4. The graph below can be considered as a cosine function. Determine the equation of this function with **smallest** possible phase shift.



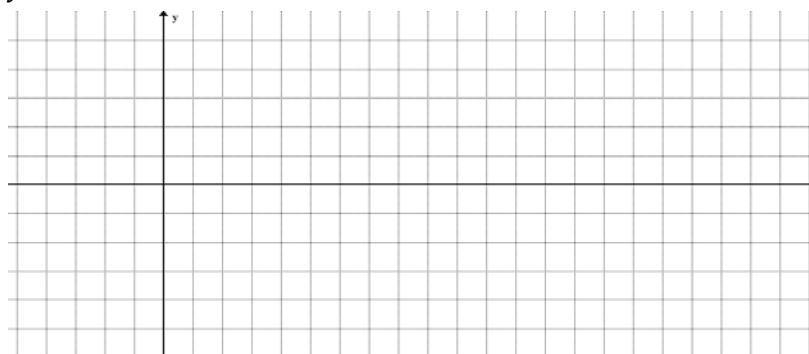
5. Each function below has either the form $y = a \sin x + d$ or $y = a \cos x + d$. Write the equation of each graph.



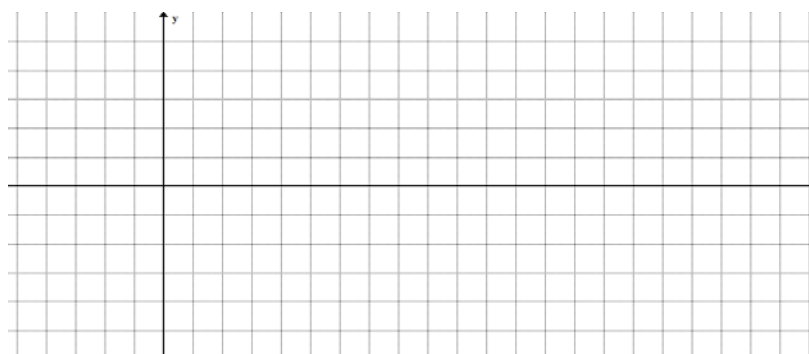


6. Graph at least one cycle of each function. Determine its domain, range, phase shift and amplitude.

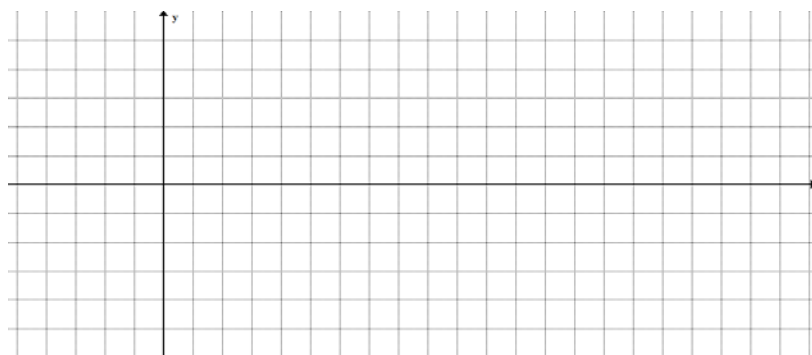
a) $y = 3 \cos x$



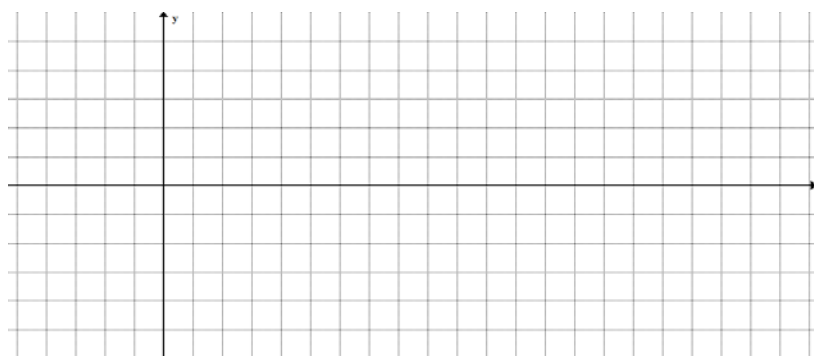
b) $y = -4 \cos x$



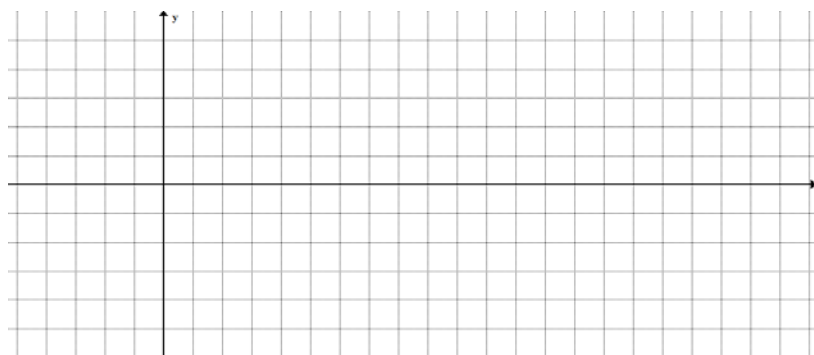
c) $y = -\cos x + 1$



d) $y = 2 \sin x - 1$



e) $y = \cos\left(x - \frac{\pi}{6}\right)$



f) $y = 5 \sin\left(x + \frac{3\pi}{4}\right)$



g) $y = -\cos\left(x + \frac{\pi}{3}\right) + 3$



h) $y = 3 \sin\left(x - \frac{5\pi}{6}\right) + 2$



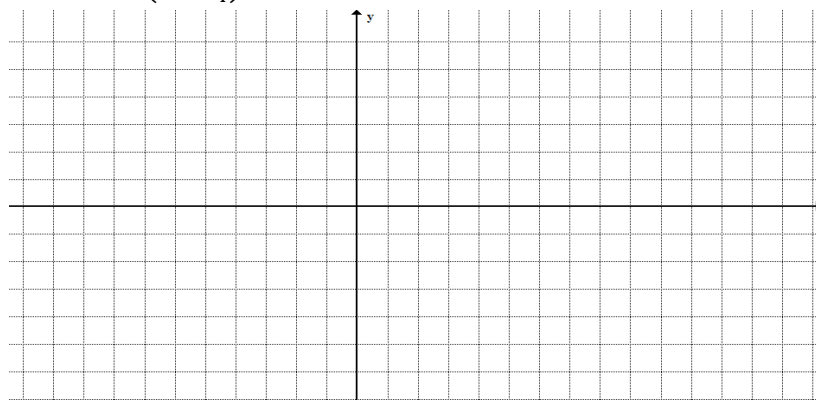
Lesson SixTransformations of Trigonometric Functions II:

1. Graph at least one cycle of each function.

a) $y = 3 \sin 4x - 1$



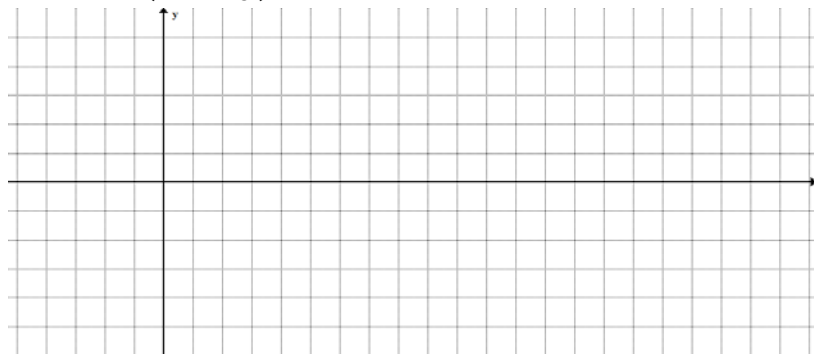
b) $y = \cos 3 \left(x + \frac{\pi}{4} \right)$



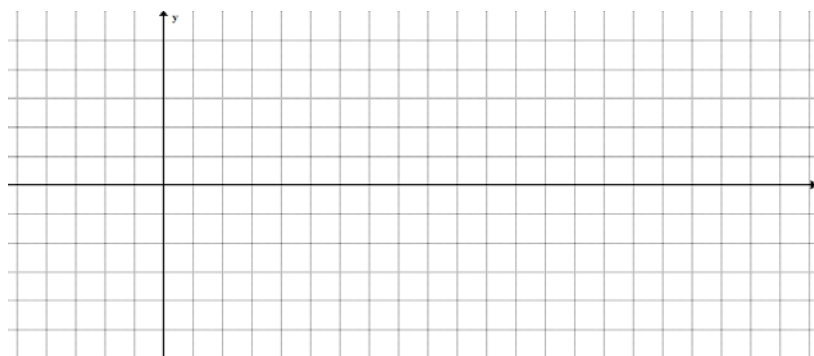
c) $y = -4 \sin 2 \left(x - \frac{\pi}{3} \right)$



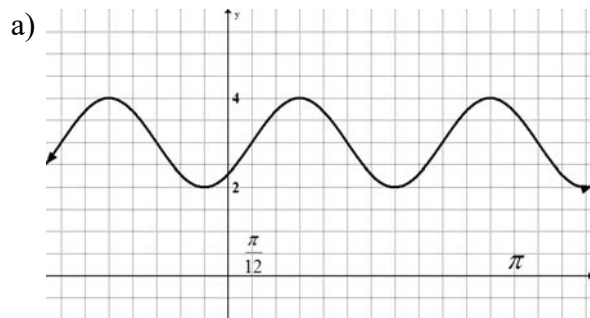
d) $y = 2 \cos\left(\frac{1}{2}x - \frac{3\pi}{8}\right)$

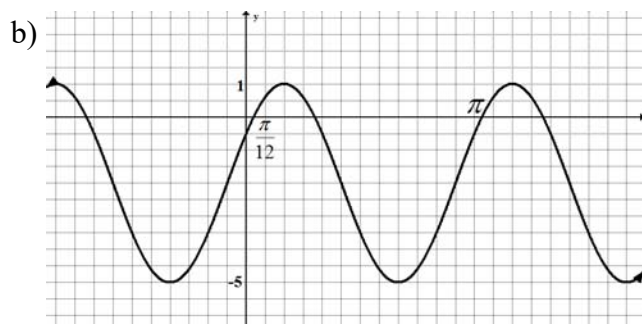


e) $y = -2\sin 2\left(x - \frac{5\pi}{6}\right) + 3$



2. Determine both the cosine and sine equation, with smallest possible phase shift, for the following graphs.





3. Write the equation of a sine curve, with smallest possible phase shift, if it has a maximum at $(\frac{\pi}{12}, 13)$ and the nearest minimum to the right at $(\frac{7\pi}{12}, 3)$.

Lesson Seven

Sinusoidal Functions with Rational Periods:

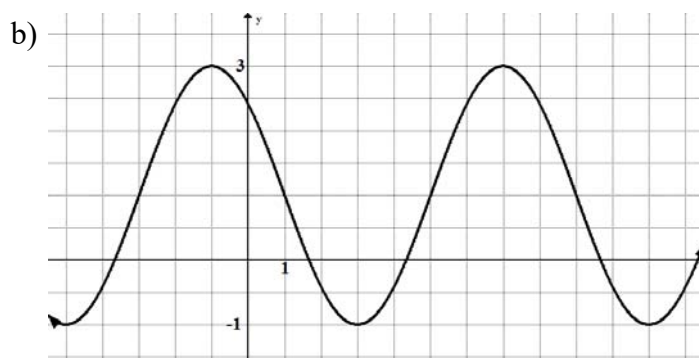
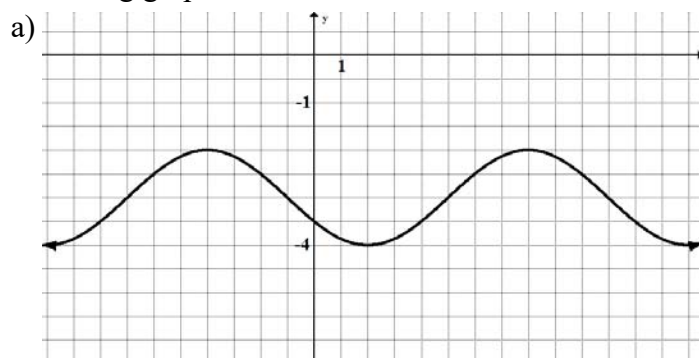
1. Given each function determine the period and phase shift.

a) $y = 2 \sin \frac{2\pi(x-3)}{5} + 1$

b) $y = -\cos \left(\frac{2\pi}{12}(x+1) \right)$

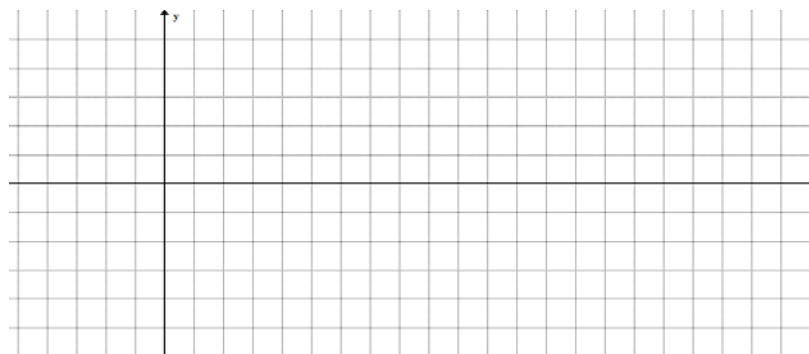
c) $y = 3 \sin \frac{\pi(x+2)}{7} - 5$

2. Determine both the cosine and sine equation, with smallest possible phase shift, for the following graphs.

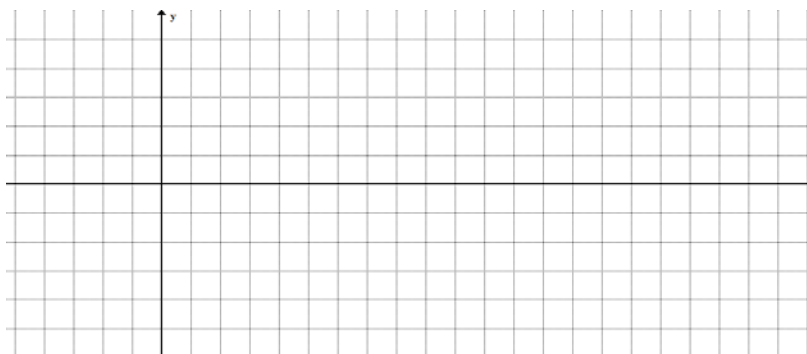


3. Graph at least one cycle of each function.

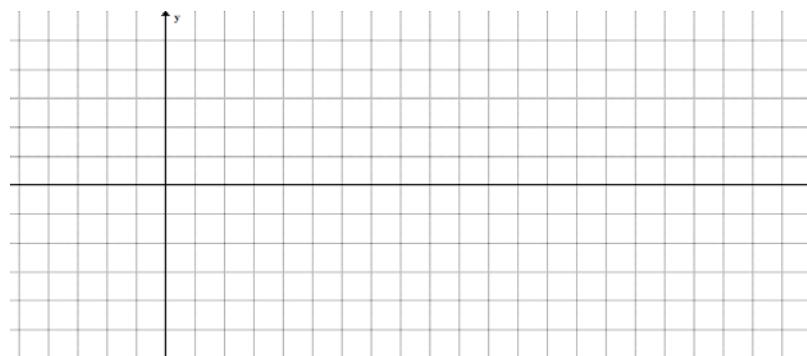
a) $y = \sin \frac{2\pi(x-5)}{12} + 1$



b) $y = 2 \cos \frac{2\pi(x+3)}{8} + 1$



c) $y = -3 \sin \frac{\pi(x+4)}{10} - 1$



d) $y = -2 \cos \frac{2\pi(x-3)}{16} + 3$



Lesson Eight

Applications of Sinusoidal Functions

1. The function $h(t) = 4.3 \sin 0.15\pi(t - 2.5) + 6.4$ gives the depth of water at a seaport, h meters, at any time, t hours, during a certain day. Use the graph of the function to find the total number of hours in the 24 hour interval starting at $t = 0$ that the depth of the water is at least 6 meters deep. (Give a rough sketch of what you graphed on your graphing calculator).
2. In an Australian city the time of sunrise can be found using the trigonometric function $T = 4.6 \sin\left(\frac{\pi}{182}(d - 6)\right) + 5$ where T is the number of hours past midnight and d is the number of days into the year.
 - a) Sketch this function.
 - b) Find algebraically the time to the nearest minute when the sun will rise on the 27th day of the year
 - c) Find how many days in the year that the sunrise is after 8 am.

3. Ferris wheel has a diameter of 45meters and its center is 43 meters off the ground. The wheel revolves once every 4 minutes. Lenny gets on the Ferris wheel at its lowest point at time zero.
- a) Write a sinusoidal equation that would model Lenny's height above the ground as a function of time.
 - b) Determine when (after how many minutes) Lenny would first be 58 meters above the ground (solve algebraically or with calculator).
4. A mass is supported by a spring so that it rests 25 cm above a table top. The mass is pulled down to a height of 10 cm above the table top and released at time $t = 0$. It takes 0.8 seconds for the mass to reach a maximum height of 40 cm above the table top. As the mass moves up and down, its height h , above the table top, is approximated by a sinusoidal function of the elapsed time t , for a short period of time. Determine a sine equation that gives the height, h , as a function of time, t .

Lesson NineGraphing the Tangent Function:

1. For each of the following state the period and the general equation for the asymptotes.

a) $y = \tan\left(x - \frac{\pi}{6}\right)$

b) $y = \tan 3x + 2$

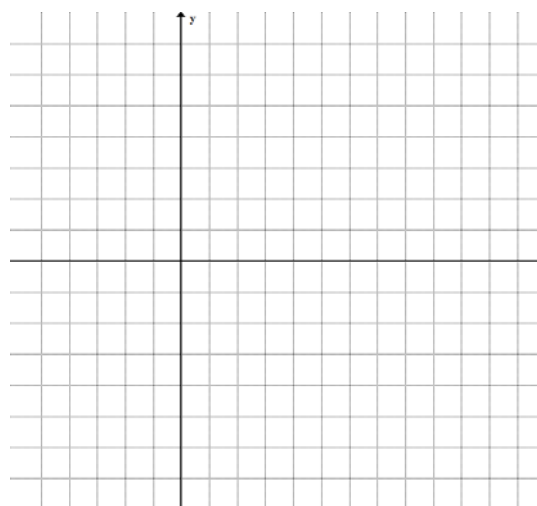
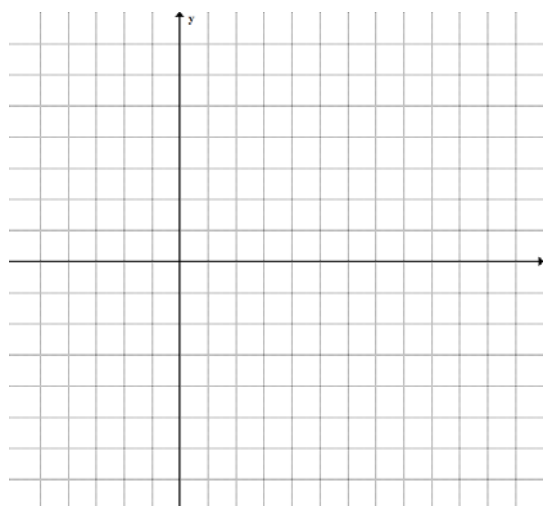
c) $y = \tan 2\left(x - \frac{\pi}{3}\right)$

d) $y = \tan \frac{\pi}{4}(x + 1) - 1$

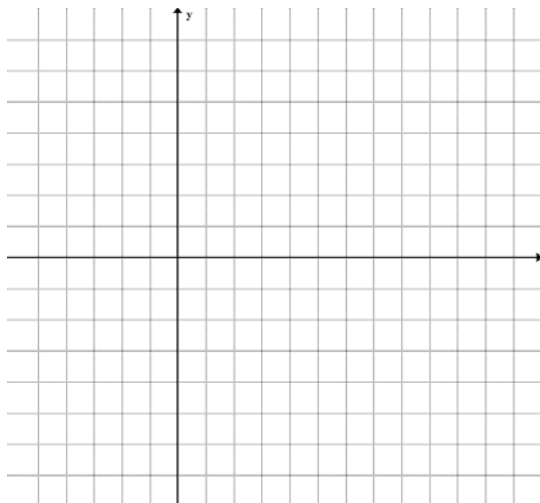
2. Graph at least two periods of each of the following functions. Clearly label the asymptotes.

a) $y = \tan 2x + 1$

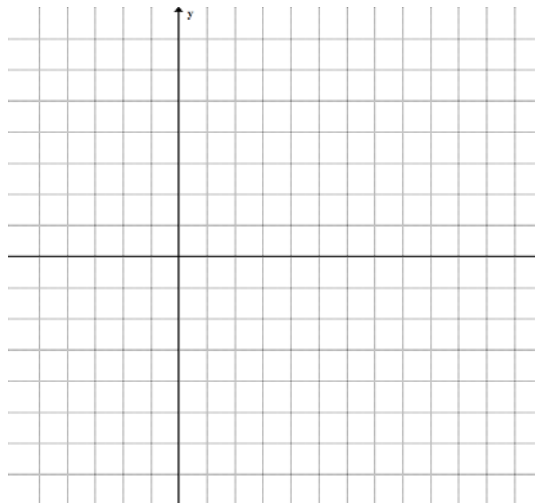
b) $y = \tan\left(x + \frac{\pi}{3}\right)$



c) $y = \tan 3\left(x - \frac{\pi}{6}\right) - 2$



d) $y = \tan \frac{\pi}{6}(x + 1)$



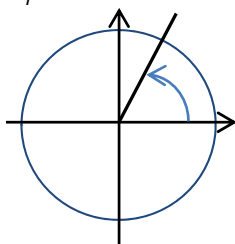
ANSWERS*Lesson One Radian Measure and Angles in Standard Position*

1. a) An angle drawn on the Cartesian plane (x - y plane), with its vertex on the origin, after rotating the desired amount and direction. b) The positive x -axis (an angle of 0) c) The line drawn for an angle in standard position.
d) An angle with a different numerical value but same standard position.
2. a) $\frac{\pi}{6}$ b) $\frac{4\pi}{3}$ c) $\frac{11\pi}{18}$ d) $-\frac{\pi}{18}$ 3. a) 60° b) 150° c) 18° d) -100°
4. a) $-220^\circ, 500^\circ$ b) $-\frac{5\pi}{7}, \frac{23\pi}{7}$ c) $-438^\circ, 282^\circ$ d) $-\frac{16\pi}{5}, \frac{4\pi}{5}$ 5. 8cm 6. 14.32cm

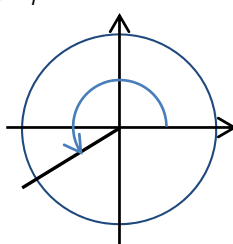
Lesson Two Sine, Cosine and Tangent

1. The smallest (acute) positive angle between the terminal arm and the x -axis.

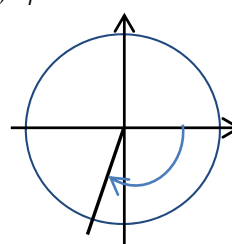
2. a) $x_r = 70^\circ$



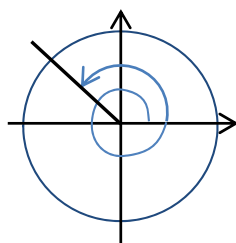
b) $x_r = 40^\circ$



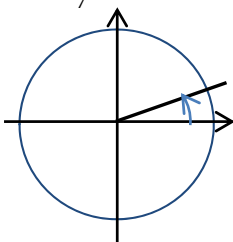
c) $x_r = 80^\circ$



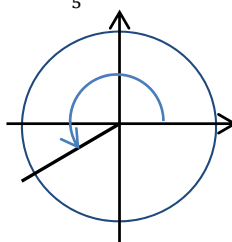
d) $x_r = 40^\circ$



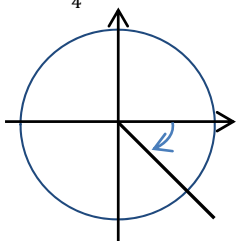
e) $\theta_r = \frac{\pi}{7}$



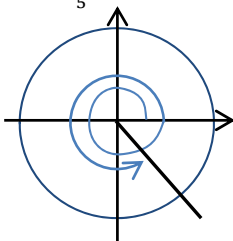
f) $\theta_r = \frac{\pi}{5}$



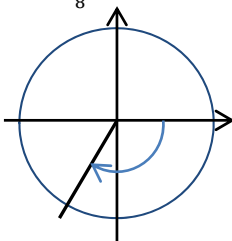
g) $\theta_r = \frac{\pi}{4}$



h) $\theta_r = \frac{2\pi}{5}$



i) $\theta_r = \frac{3\pi}{8}$



3. a) $\sin \theta = 0.8, \cos \theta = -0.6, \tan \theta = -\frac{4}{3}$

b) $\sin \theta = -0.6, \cos \theta = 0.8, \tan \theta = -\frac{3}{4}$

4. a) $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$

5. $\cos \theta = -\frac{\sqrt{35}}{6}, \tan \theta = -\frac{1}{\sqrt{35}}$

6. a) $\sin \theta = -\frac{5}{\sqrt{29}}, \cos \theta = -\frac{2}{\sqrt{29}}, \tan \theta = \frac{5}{2}$

b) $\sin \theta = \frac{7}{\sqrt{58}}, \cos \theta = -\frac{3}{\sqrt{58}}, \tan \theta = -\frac{7}{3}$

7. $\sin \theta = -\frac{\sqrt{5}}{3}$

8. $\tan \theta = \frac{1}{\sqrt{15}}$

9. $\cos \theta = -\sqrt{1-a^2}$

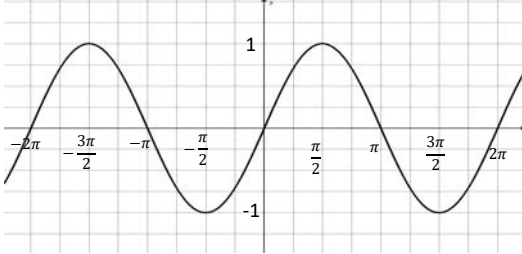
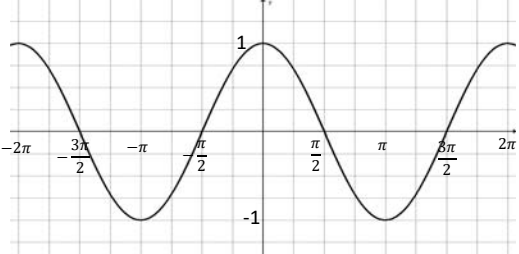
10. $\cos \theta = -\frac{2}{\sqrt{13}}$

11. $\sin(\pi + \theta) = -\frac{\sqrt{7}}{4}$

Lesson Three Sine, Cosine and Tangent of Angles

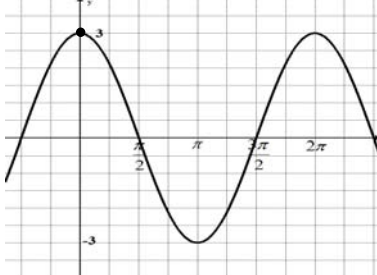
1. It is a mnemonic that helps us remember which trigonometric ratios are positive in which quadrants.
2. a) -0.304 b) -2.748 c) -47.196 d) 2.119
3. a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{\sqrt{2}}$ c) $\frac{1}{\sqrt{3}}$ d) $-\frac{1}{\sqrt{2}}$ e) 0 f) $\sqrt{2}$ g) $-3\sqrt{3}$ h) $-\frac{1}{\sqrt{2}}$ i) $-\sqrt{3}$
 j) $\frac{8\sqrt{3}}{3}$ k) $-\frac{16}{\sqrt{3}}$ l) $\frac{-3\sqrt{2}}{2}$ m) 2 n) 0 o) $\frac{1}{2}$ p) $-\frac{1}{2}$ q) $-\frac{1}{2}$ r) 1
4. a) Quadrant II b) Quadrant IV

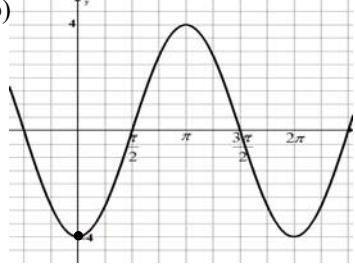
Lesson Four Graphing Sine and Cosine Functions

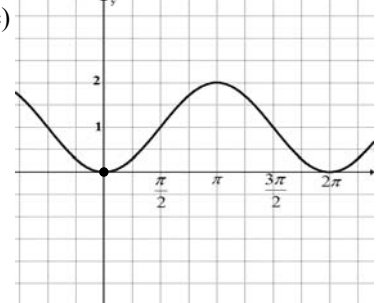
1. a)  b) 
2. a) max value = 1, occurs at $\theta = \frac{\pi}{2}$ b) min value = -1 , occurs at $\theta = \frac{3\pi}{2}$ c) $\theta = 0, \pi, 2\pi$
3. a) max value = 1, occurs at $\theta = 0, 2\pi$ b) min value = -1 , occurs at $\theta = \pi$ c) $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

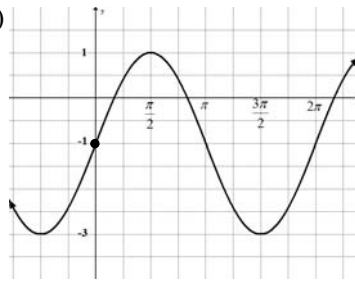
Lesson Five Transformations of Trigonometric Functions I

1. a) The 'a' value changes both the amplitude as well as how the graph "starts"
 b) The 'c' value produces a phase shift c) The 'd' value changes the location of the central axis
2. a) $y = \sin(x - 4)$ b) $y = 3 \sin x - 1$ c) $y = \frac{1}{2} \sin\left(x + \frac{\pi}{7}\right)$
3. $y = \sin\left(x - \frac{\pi}{6}\right)$ 4. $y = \cos\left(x + \frac{\pi}{4}\right)$ 5. a) $y = 2 \cos x + 1$ b) $y = 1.5 \sin x - 2$
 c) $y = -2 \cos x$ d) $y = -\sin x - 2$

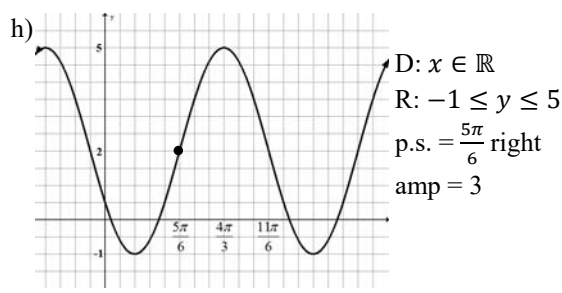
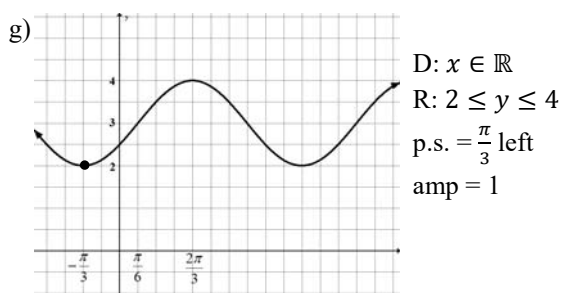
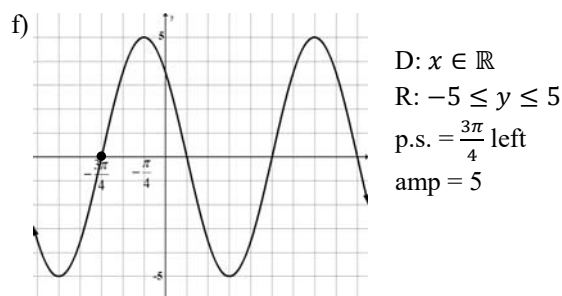
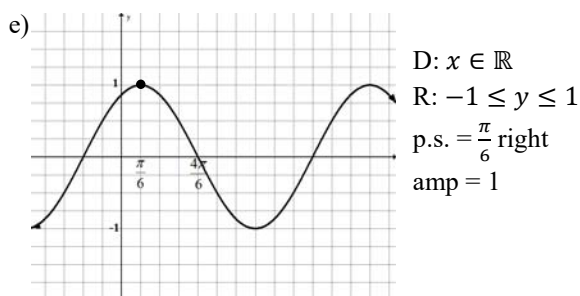
6. a) 

D: $x \in \mathbb{R}$
 R: $-3 \leq y \leq 3$
 p.s. = 0
 amp = 3
- b) 

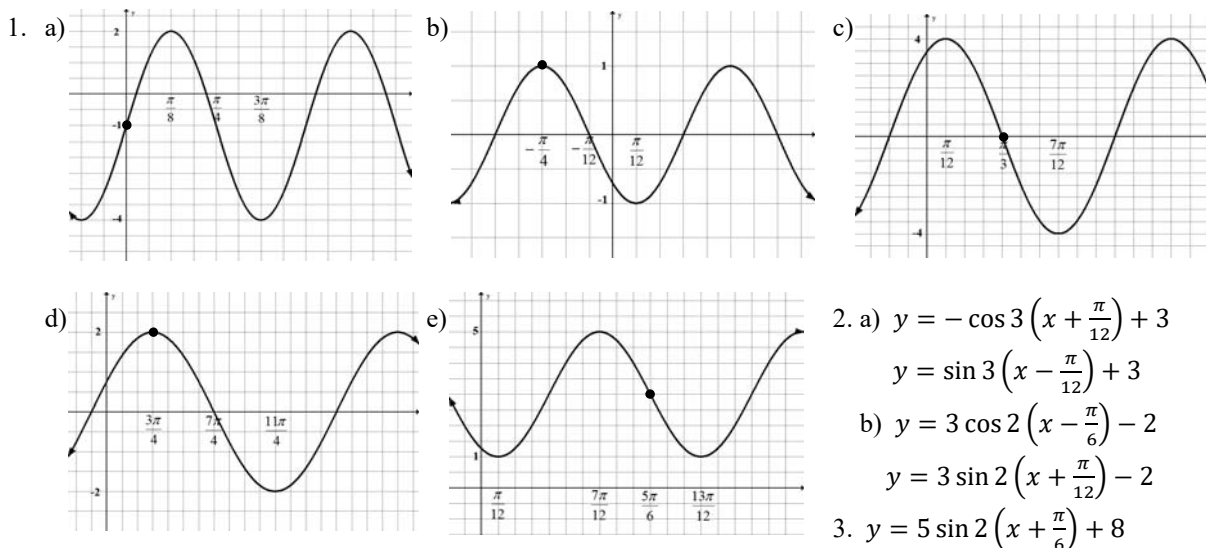
D: $x \in \mathbb{R}$
 R: $-4 \leq y \leq 4$
 p.s. = 0
 amp = 4
- c) 

D: $x \in \mathbb{R}$
 R: $0 \leq y \leq 2$
 p.s. = 0
 amp = 1
- d) 

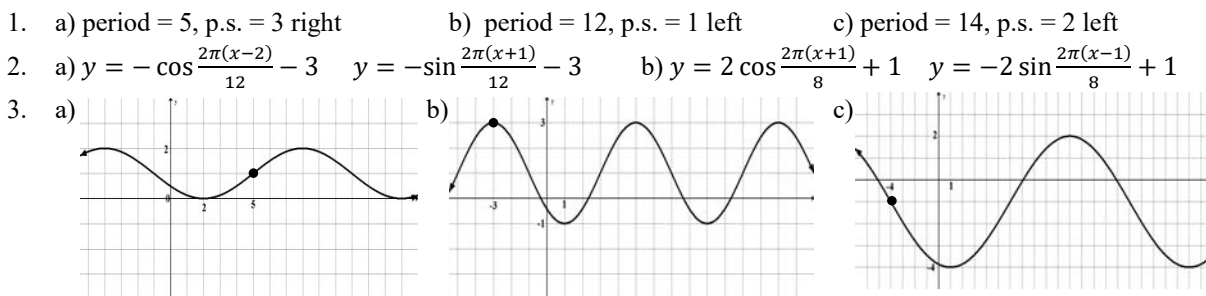
D: $x \in \mathbb{R}$
 R: $-3 \leq y \leq 1$
 p.s. = 0
 amp = 2

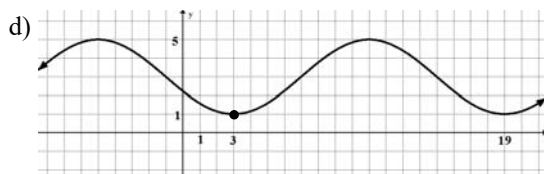


Lesson Six Transformations of Trigonometric Functions II

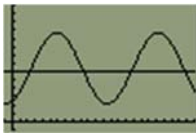


Lesson Seven Sinusoidal Functions with Rational Periods

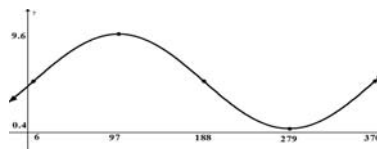




Lesson Eight Applications of Sinusoidal Functions

1. a)  14.12 hours

2. a)



b) 6:38 am

c) 100 days

3. a) $h(t) = -22.5 \cos\left(\frac{2\pi t}{4}\right) + 43$ or $h(t) = 22.5 \sin\left(\frac{2\pi(t-1)}{4}\right) + 43$

b) 1.46mins

4. a) $h(t) = 15 \sin\left(\frac{2\pi(t-0.4)}{1.6}\right) + 25$

Lesson Nine Graphing the Tangent Function

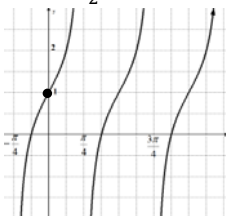
1. a) period: π asymptote: $x = \frac{2\pi}{3} + \pi n$

b) period: $\frac{\pi}{3}$ asymptote: $x = \frac{\pi}{6} + \frac{\pi}{3}n$

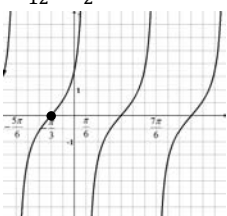
c) period: $\frac{\pi}{2}$ asymptote: $x = \frac{7\pi}{12} + \frac{\pi}{2}n$

d) period: 4 asymptote: $x = 1 + 4n$

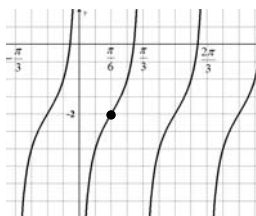
2. a)



b)



c)



d)

